

Decoherence-Free Emergence of Macroscopic Local Realism for entangled photons in a cavity

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Abstract

We investigate the influence of environmental noise on polarization entangled light generated by parametric emission in a cavity. By adopting a recently developed separability criterion, we show that: i) self-stimulation may suppress the detrimental influence of noise on entanglement; ii) when self-stimulation becomes effective, a classical model of parametric emission incorporating noise provides the same results of quantum theory for the expectation values involved in the separability criterion. Moreover we show that, in the macroscopic limit, it is impossible to observe violations of local realism with measurements of n -particle correlations, whatever n but finite. These results provide an interesting example of the emergence of macroscopic local realism in the presence of strong entanglement even in the absence of decoherence.

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Entanglement is one of the most profound features of quantum mechanics. It plays an essential role in all branches of quantum information theory [1]. Bell theorem [2], which is derived from Einstein-Podolsky-Rosen's (EPR's) notion of local realism [3], quantifies how measurements on entangled quantum mechanical systems may invalidate local classical models of reality. While all bipartite pure entangled states violate some Bell inequality [4], the relationship between entanglement and non-locality for mixed quantum states is not well understood yet [5, 6]. Moreover recent proposals [7] and realizations [8, 9, 10, 11, 12] of many-particle entangled quantum states require a better understanding of the domain of validity of quantum behaviour. A relevant point is whether the conflict between classical elements of reality and quantum mechanics may persist at a macroscopic level [13]. In particular continuous-variable entanglement of intense light sources has been recently demonstrated in [8, 10], and polarization entanglement of macroscopic beams in [11]. Moreover, it has been recently shown that a source of strongly entangled states with photon numbers up to a million seems achievable [7]. In these works entanglement has been tested and quantified by using specific separability criteria. They consist in inequalities among expectation values of experimentally measurable quantities, that are violated by entangled quantum states [14]. It is worth noting that there is a profound conceptual difference between Bell inequalities and separability (or entanglement) criteria. The violation of the former implies quantum features in conflict with local realism of classical mechanics. Whereas the latter have been derived assuming from the beginning that we are dealing with a quantum system. Specifically demonstrations of these inseparability criteria (see e.g. [7, 14]) exploit the fact that involved operators do not commute. The behaviour of entanglement towards a macroscopic situation (even close to classical every-day life phenomena) and its robustness versus noise and decoherence are not well understood yet. In this Letter we shall address this crucial problem focusing on a particular very promising source of macroscopic entanglement: parametric down-conversion of photons inside an optical cavity. On the one hand we shall quantify the detrimental influence of such environment channels and show how self-stimulation may suppress them efficiently. On the other hand we shall tackle the problem of the macroscopic limit and of the emergence of classical elements of reality within a quantum framework.

In order to investigate the relationship among entanglement, quantum nonlocality, and the macroscopic limit, we adopt a weaker nonlocality concept [15] based on an *adherence*

on the physical system. In particular we do not ask quantum correlations to exceed bounds that cannot be violated by classical correlations, but we limit us to compare the quantum findings with the corresponding classical stochastic calculations for the specific physical system under investigation. However, a classical model endowed with stochastic noise represents a realization of a local stochastic hidden variable theory and the customary lines of thoughts are recovered. To this end we consider polarization entangled light from parametric down-conversion driven by an intense pump field inside a cavity. The multiphoton states produced are close approximations to singlet states of two very large spins [7]. The interaction Hamiltonian describing the process is given by

$$\hat{H} = i\hbar\Omega(\hat{a}_h^\dagger\hat{b}_v^\dagger - \hat{a}_v^\dagger\hat{b}_h^\dagger) + H.c., \quad (1)$$

where a and b refer to the two conjugate directions along which the frequency-degenerate photon pairs are emitted. h and v denote horizontal and vertical polarization and $\hbar\Omega$ is a coupling constant whose magnitude depends on the nonlinear coefficient of the crystal and on the intensity of the pump pulse. In the absence of losses, within the Heisenberg picture, the interaction Hamiltonian in (1) dictates the following time evolution of photon operators:

$$\begin{aligned} \hat{a}_{h,v} &= \hat{a}_{h,v}(0) \cosh(r) \pm \hat{b}_{v,h}^\dagger(0) \sinh(r) \\ \hat{b}_{v,h} &= \hat{b}_{v,h}(0) \cosh(r) \pm \hat{a}_{h,v}^\dagger(0) \sinh(r), \end{aligned} \quad (2)$$

where the interaction parameter r is $\Omega\tau$ being $\tau = L/v$ the interaction time interval. i.e. the time spent by the photons with velocity v inside a crystal of length L . In the absence of losses and considering the photon vacuum as initial state, the Hamiltonian (1) produces a multiphoton quantum state $|\psi\rangle$ that is the polarization equivalent of a spin singlet state, where the spin components correspond to the Stokes polarization parameters, $\hat{J}_z^A = (\hat{a}_h^\dagger\hat{a}_h - \hat{a}_v^\dagger\hat{a}_v)/2$, $\hat{J}_x^A = (\hat{a}_+^\dagger\hat{a}_+ - \hat{a}_-^\dagger\hat{a}_-)/2$, and $\hat{J}_y^A = (\hat{a}_l^\dagger\hat{a}_l - \hat{a}_r^\dagger\hat{a}_r)/2$, where $\hat{a}_{+,-} = (\hat{a}_h \pm \hat{a}_v)/\sqrt{2}$ correspond to linearly polarized light at $\pm 45^\circ$, and $\hat{a}_{l,r} = (\hat{a}_h \pm i\hat{a}_v)/\sqrt{2}$ to left- and right-ended circularly polarized light. The label A refers to the a modes. Analogous expressions can be obtained for $\hat{\mathbf{J}}^B$ in terms of the b modes. It has been shown [7] that the state $|\psi\rangle$ is a singlet state of the total angular momentum operator $\hat{\mathbf{J}} = \hat{\mathbf{J}}^A + \hat{\mathbf{J}}^B$. As a consequence $\langle \psi | \hat{\mathbf{J}}^2 | \psi \rangle = 0$. Losses, fluctuations and imperfections can lead to nonzero values for the total angular momentum, corresponding to nonperfect correlations between the Stokes parameters in the a and b beams. Within this picture it is straightforward to include the

presence of noise in the system assuming that before the pump is switched on the system is in an incoherent thermal-like state described by a diagonal density matrix. Dealing with such systems the first analysis one may perform is an intensity measurement:

$$n_{ah(v)}(r) \equiv \langle \hat{a}_{h,v}^\dagger \hat{a}_{h,v} \rangle = \sinh^2 r + n_o(1 + 2 \sinh^2 r), \quad (3)$$

where $n_o \equiv \langle \hat{a}_{h,v}^\dagger(0) \hat{a}_{h,v}(0) \rangle = \langle \hat{b}_{h,v}^\dagger(0) \hat{b}_{h,v}(0) \rangle$ is the noise present in the system. It is easy to identify two different contributions: the first term arises from vacuum fluctuations and describes true (eventually self-stimulated) spontaneous emission, vanishing in the absence of parametric interaction. The last term in Eq. (3) describes a classical-like amplification of the input thermal noise n_o . It is worth noting that the solution of the corresponding classical model of the optical parametric amplifier has the same structure of Eq. (2) with a and b being of course replaced by classical amplitudes [16]:

$$n^C(r) \equiv \langle a_{h,v}^* a_{h,v} \rangle = n_o^C(1 + 2 \sinh^2 r), \quad (4)$$

where $\langle \rangle$ denotes statistical average and n_o^C describes, as before, statistical noise. For small r values and for negligible n_o ($n_o \ll r^2$), quantum and classical descriptions lead to distinct functions of r , being $n(r) \simeq r^2$ and $n^C(r) \simeq n_o^C(0)$. This distinct behaviour can be related to the fact that vacuum fluctuations (in contrast to classical ones) do not produce work and hence, while they can stimulate pump scattering, they cannot be directly evidenced by photodetection. In contrast when r increases, it is no more possible to distinguish quantum behaviour by means of simple intensity measurements. In particular for $r \geq 2$ a classical model with $n_o^C = n_o + 1/2$ (in order to properly include vacuum fluctuations), is able to give an intensity description that cannot be distinguished from the quantum one. It is worth noting that this behaviour can also be found in intriguing second-order interference effects [15, 17] and it agrees with the old idea that many quanta in a system give rise to a classical-like behaviour. Other relevant second order correlations are given by the following anomalous correlators:

$$\begin{aligned} A_{hv(vh)} &= \langle \hat{a}_{h(v)} \hat{b}_{v(h)} \rangle = (n_o + 1/2) \sinh(2r) \\ A_{hv(vh)}^C &= \langle a_{h(v)} b_{v(h)} \rangle = n_o^C \sinh(2r), \end{aligned} \quad (5)$$

which quantify the pair correlation induced by the parametric process. Equation (5) shows that replacing again $n_o^C = n_o + 1/2$, the classical description coincides with the quantum one.

If the above considerations can be formulated for second-order correlations, now we want to focus our attention on what we can say about entanglement measurements on this system. In order to test the degree of entanglement of this correlated quantum system, a simple inseparability criterion has been derived [7]: if $\langle \hat{\mathbf{J}}^2 \rangle / \langle \hat{N} \rangle$ (where $\hat{N} = \hat{N}_A + \hat{N}_B$ is the total photon number) is smaller than $1/2$, then the state is entangled. We now consider the system at $r \leq 0$ (before switching on the pump) to be in thermal equilibrium. In particular we assume the system at $r \leq 0$ to be in a completely incoherent (mixed) state described by a diagonal density matrix. The only input for the system are thermal noise (if $T \neq 0$) and vacuum fluctuations. By using Eq. (2) and the density matrix for thermal equilibrium, we obtain:

$$\frac{\langle \hat{\mathbf{J}}^2 \rangle}{\langle \hat{N} \rangle} = \frac{3n_0(n_0 + 1)}{4n_0 + (1 + 5n_0) \sinh^2 r}. \quad (6)$$

At zero temperature $n_0 = 0$ and the system is maximally entangled ($\langle \hat{\mathbf{J}}^2 \rangle = 0$) independently of the magnitude of r . Eq. (6) also shows that, even when thermal noise largely exceeds vacuum fluctuations ($n_0 \gg 1$), $\langle \hat{\mathbf{J}}^2 \rangle / \langle \hat{N} \rangle$ goes below $1/2$ provided that r is sufficiently large. Moreover $\langle \hat{\mathbf{J}}^2 \rangle / \langle \hat{N} \rangle \rightarrow 0$ for $r \rightarrow \infty$. This result shows that macroscopic entanglement may in principle be achieved even in the presence of strong fluctuations, provided that self-stimulation of the emitted pairs takes place. In particular the system becomes entangled when $\sinh^2 r > 2n_0(3n_0 + 1)/(5n_0 + 1)$. Nevertheless, according to the criterion, in order to beat the detrimental effect of strong fluctuations on entanglement we need to rely on self-stimulation. Generally speaking it is known that entanglement as well as violations of Bell inequalities have limited resistance to noise. In the present case a small amount of noise is sufficient to completely destroy entanglement, e.g. for $r = 10^{-3}$, $n_0 = 2 \times 10^{-6}$ is sufficient to wash out entanglement according to Eq. (6); nevertheless it is sufficient to start self-stimulation (by increasing r) to restore it. In order to get a deeper insight we seek some additional information wondering if, from the criterion viewpoint (this time), we can distinguish between classical and quantum findings. To this end we put the two descriptions (classical and quantum) on equal footing and compute the entanglement criterion evaluating their differences and similarities. In particular, a classical calculation, computed according to the above described prescriptions, gives $\langle \mathbf{J}^2 \rangle / \langle N \rangle = 3n_0^C/(4 + 5 \sinh^2 r)$. Of course classical optics does not require a minimum amount of fluctuations, thus within a classical model it is possible to obtain $\langle \mathbf{J}^2 \rangle / \langle N \rangle < 0.5$. In the low excitation regime ($r \ll 1$) and n_0 lower than

r^2 classical and quantum calculations of $\langle \hat{\mathbf{J}}^2 \rangle / \langle \hat{N} \rangle$ display very different variations with r as it happens for simpler intensity cases. Moreover the classical calculation above provides a result that as Eq. (6) goes to zero for r much larger than n_0 . This means that experiments eventually demonstrating macroscopic entanglement for this system can be accounted for in terms of purely classical correlations, with no need for a quantum-mechanical explanation. Analogous conclusions can be reached for different experimentally tested criteria [8, 10, 11, 12]. This does not mean at all that the entanglement criterion is wrong or contradictory, neither that macroscopic entangled systems cannot display quantum nonlocality effects. Recently it has been shown that quantum states of a nondegenerate optical parametric amplifier display their maximum violation of the Bell inequality due to Clauser, Horne, Shimony, and Holt just in the macroscopic limit ($r \rightarrow \infty$) [18]. However the above analysis suggests that there is a large class of quantum correlation functions that cannot differ from the corresponding classical ones in the macroscopic limit. Indeed we can define the following set of correlation functions $\langle \hat{\mathcal{B}}_\alpha^{(n)\dagger} \hat{\mathcal{B}}_{\alpha'}^{(n')} \rangle$, where $\hat{\mathcal{B}}_{m,l,k}^{(n)} = (\hat{b}_v)^{n-m}(\hat{b}_h)^{m-l}(\hat{a}_v)^{l-k}(\hat{a}_h)^k$ is a generic n -particle destruction operator. These correlation functions (and also their classical counterparts $\langle \mathcal{B}_\alpha^{(n)*} \mathcal{B}_{\alpha'}^{(n')} \rangle$) are different from zero only if $n = n'$ and $\alpha \equiv (k, l, m) = \alpha'$. Since we are dealing with a Gaussian system [19], such correlators (as well as their classical counterparts) can be decomposed in a sum of products of second order correlation functions only ($n(r)$ and $A(r)$). Since $A(r) = A^C(r)$ (for $n_0^C = n_0 + 1/2$), and $n(r)/n^C(r) \rightarrow 1$ for $r \rightarrow \infty$, we obtain that

$$\lim_{r \rightarrow \infty} \frac{\langle \hat{\mathcal{B}}_\alpha^{(n)\dagger} \hat{\mathcal{B}}_\alpha^{(n)} \rangle}{\langle \mathcal{B}_\alpha^{(n)*} \mathcal{B}_\alpha^{(n)} \rangle} = 1. \quad (7)$$

This result implies that *it is impossible to observe violations of macroscopic local realism (e.g. violations of Bell inequalities, including those which are not yet known) by measurements of any finite set of expectation values that can be expanded as a finite sum of these correlation functions*. It is easy to verify via explicit calculations that convergence of limit (7) is very fast even for large values of n . Bell inequalities can be schematically expressed as $\mathcal{F}(\langle \mathcal{B}_\alpha^{(n)*} \mathcal{B}_\alpha^{(n)} \rangle(r)) \leq \mathcal{L}(n)$, where $\mathcal{L}(n)$ is a bound imposed by local realism that cannot be violated by classical correlations, and \mathcal{F} is a generic continuous function of $\langle \mathcal{B}_\alpha^{(n)*} \mathcal{B}_\alpha^{(n)} \rangle$ depending also on the different settings chosen by the observers. From Eq. (7): $\mathcal{F}(\langle \hat{\mathcal{B}}_\alpha^{(n)\dagger} \hat{\mathcal{B}}_{\alpha'}^{(n')} \rangle(r)) \rightarrow \mathcal{F}(\langle \mathcal{B}_\alpha^{(n)*} \mathcal{B}_{\alpha'}^{(n')} \rangle(r))$ when $r \rightarrow \infty$, thus any bound \mathcal{L} cannot be violated (in the limit). One example of these wide class of Bell inequalities can be found in

[20]. These results can be generalized to multipartite situations obtained e.g. by inserting in the setup a number of beamsplitters [21].

So far we have considered a situation where the system is initially in thermal equilibrium, but eq. (2) has been obtained under the hypothesis that the system has no losses, hence it is assumed that for the steady-state calculations at any value of the interaction parameter r the system is disconnected from the environment. However in real systems amplification, losses, and noise disturbances actually happen simultaneously. In the presence of losses, the (steady-state) Heisenberg equations have to be replaced by (t-dependent) Langevin equations with noise sources. In the symmetric case (equal losses for all the four modes), we obtain,

$$\begin{aligned}\hat{a}_h(t) = & \hat{a}_h(0) \cosh \Delta(t, 0) + \hat{b}_v^\dagger(0) \sinh \Delta(t, 0) \\ & + \int_0^t e^{-\lambda(t-t')} \cosh \Delta(t, t') \hat{f}_{ah}(t') dt' \\ & + \int_0^t e^{-\lambda(t-t')} \sinh \Delta(t, t') \hat{f}_{bv}^\dagger(t') dt'\end{aligned}\quad (8)$$

where $\Delta(t, t') = \int_{t'}^t \kappa(t'') dt'' = \frac{\kappa_0}{\Lambda}(e^{-\Lambda t'} - e^{-\Lambda t})$; $\hat{f}_\alpha(t)$ are Bose quantum noise operators associated with the losses [22] (α denotes the specific mode e.g. $\alpha \equiv (a, h)$). In the following we will assume $\langle \hat{f}_\alpha^\dagger(t) \hat{f}_{\alpha'}(t') \rangle = n_0 \delta_{\alpha, \alpha'} \delta(t - t')$. Analogous expressions can be obtained for the other three modes. Fig. 1 displays the quantum and the classical calculation of

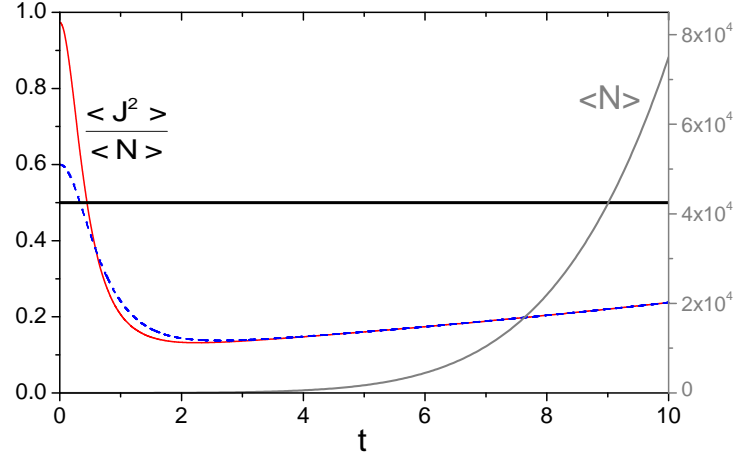


FIG. 1: (color online) Time development of $\langle \mathbf{J}^2 \rangle / \langle N \rangle$ according to classical (dashed line) and quantum (continuous line) mechanics, and quantum calculation of the total mean photon number $\langle N \rangle$. Parameters are given in the text.

$\langle \hat{\mathbf{J}}^2 \rangle / \langle N \rangle$ for $\lambda = \Lambda = 0.1$, $\kappa_0 = 1$. For the quantum (continuous line) and the classical (dashed line) calculations we adopted $n_0 = 0.3$ and $n_0^{(cl)} = 0.8$ respectively. The figure clearly shows that for $r > 3$ classical and quantum results cannot be distinguished. These results show that i) also in this more realistic case self-stimulation can suppress the detrimental effect of noise ii) coincidence between classical and quantum results can be obtained without requiring $n \gg 1$ if we choose $n_0^{(cl)} = n_0 + 1/2$; iii) the decoherence due to losses and noise seems to affect equally quantum entanglement and classical correlations, hence it cannot be invoked in the present case for the emergence of a classical behaviour. In order to interpret our results, we distinguish between two situations: When $r \ll 1$, the probability that the system gives rise to states with more than two-photons is negligible, so measurements of $\langle \hat{\mathbf{J}}^2 \rangle$ and of $\langle \hat{N} \rangle$ can probe the system at a microscopic level, but with a lot of particles (when r increases) both $\langle \hat{\mathbf{J}}^2 \rangle$ and $\langle \hat{N} \rangle$ become macroscopic observables unable to probe the system fluctuations with precision at a few quanta level. In this case the information recovered by observations is a coarse grained quantity missing the underlying quantum structure. This lack of microscopic information seems able to introduce elements of local realism even in the presence of strong entanglement and in the absence of decoherence. Of course the lack of information here described should not be confused with a lack of precision of measurements, here assumed with unlimited precision. As shown in Ref. [23], the partition of a quantum system into subsystems and hence the entanglement structure, is dictated by the set of operationally accessible interactions and measurements. A given set can hide a multipartite structure. Our results suggest that, analogously, the set of operationally accessible measurements and their ability to catch the quantized structure of the system, determines the quantum or classical nature of the observed correlations. As pointed out above, our findings (included Eq. (7)) do not imply that macroscopic entangled systems cannot display quantum nonlocality effects. As already mentioned, it has been shown recently that these kind of systems do violate CHSH Bell inequality just in the microscopic limit ($r \rightarrow \infty$) and hence do display nonlocality features. However in that case the Bell operator is constructed by means of operators with single quantum sensitivity independently of the number of particles in the system in contrast to operators $\hat{\mathcal{B}}_\alpha^{(n)\dagger} \hat{\mathcal{B}}_\alpha^{(n)}$. Of course these operators cannot be expanded in a finite sum of operators $\hat{\mathcal{B}}_\alpha^{(n)\dagger} \hat{\mathcal{B}}_\alpha^{(n)}$.

The example of the emergence of macroscopic local realism in the presence of strong entanglement, shown here, provides insight into the boundary between the classical and

quantum world. These results, with the care that they have been obtained for a Gaussian system, despite the feasible realization of systems with a huge amount of entangled particles, suggest that the lack of information gathered by coarse-grained observations may lead to the introduction of elements of local realism even in the presence of strong entanglement and in the absence of decoherence. In particular, Eq. (7) shows that violations of local realism for macroscopic Gaussian states are impossible with an apparatus able to measure finite-order correlations. Further investigations are needed to understand if the results here presented for entangled Gaussian systems can be extended to more general quantum systems.

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